(15.1a) “Associativity of +”: (a + b) + c = a + (b + c)

(15.1b) “Associativity of ·”: (a · b) · c = a · (b · c)

(15.2a) “Symmetry of +”: a + b = b + a

(15.2b) “Symmetry of ·”: a · b = b · a

(15.3a) “Additive identity” “Identity of +”: 0 + a = a

(15.3b) “Additive identity” “Identity of +”: a + 0 = a

(15.4a) “Multiplicative identity“Identity of ·”: 1 · a = a

(15.4b) “Multiplicative identity“Identity of ·”: a · 1 = a

(15.5a) “Distributivity of · over +”: a · (b + c) = a · b + a · c

(15.5b) “Distributivity of · over +”: (b + c) · a = b · a + c · a

(15.9) “Zero of ·”: a · 0 = 0

(15.13) “Unary minus”: a + (- a) = 0

(15.14) “Subtraction”: a - b = a + (- b)

(15.17) “Self-inverse of unary minus”: - (- a) = a

(15.19) “Distributivity of unary minus over +”: -(a + b) = (- a) + (- b)

(15.24) “Right-identity of -”: a - 0 = a

(15.25a) “Mutual associativity of + and -”: a + (b - c) = (a + b) - c

(15.25b) “Subtraction of addition”: a - (b + c) = (a - b) - c

(15.29a) “Distributivity of · over -”: (a - b) · c = a · c - b · c

(15.29b) “Distributivity of · over -”: c · (a - b) = c · a - c · b

(3.12) “Double negation”: ¬ ¬ p ≡ p

(3.8) “Definition of `false`”: false ≡ ¬ true

(3.13) “Negation of `false`”: ¬ false ≡ true

(3.24) “Symmetry of ∨”: p ∨ q ≡ q ∨ p

(3.25) “Associativity of ∨”: (p ∨ q) ∨ r ≡ p ∨ (q ∨ r)

(3.26) “Idempotency of ∨”: p ∨ p ≡ p

(3.29) (3.29a) “Zero of ∨”: p ∨ true ≡ true

(3.29) (3.29b) “Zero of ∨”: true ∨ p ≡ true

(3.30) (3.30a) “Identity of ∨”: p ∨ false ≡ p

(3.30) (3.30b) “Identity of ∨”: false ∨ p ≡ p

(3.28) “Excluded middle” “LEM”: p ∨ ¬ p ≡ true

(3.36) “Symmetry of ∧”: p ∧ q ≡ q ∧ p

(3.37) “Associativity of ∧”: (p ∧ q) ∧ r ≡ p ∧ (q ∧ r)

(3.38) “Idempotency of ∧”: p ∧ p ≡ p

(3.39) (3.39a) “Identity of ∧”: p ∧ true ≡ p

(3.39) (3.39b) “Identity of ∧”: true ∧ p ≡ p

(3.40) (3.40a) “Zero of ∧”: p ∧ false ≡ false

(3.40) (3.40b) “Zero of ∧”: false ∧ p ≡ false

(3.42) “Contradiction”: p ∧ ¬ p ≡ false

(3.47) (3.47a) “De Morgan”: ¬ (p ∧ q) ≡ ¬ p ∨ ¬ q

(3.47) (3.47b) “De Morgan”: ¬ (p ∨ q) ≡ ¬ p ∧ ¬ q

(3.45) (3.45a) “Distributivity of ∨ over ∧”: p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

(3.45) (3.45b) “Distributivity of ∨ over ∧”: (q ∧ r) ∨ p ≡ (q ∨ p) ∧ (r ∨ p)

(3.46) (3.46a) “Distributivity of ∧ over ∨”: p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)

(3.46) (3.46b) “Distributivity of ∧ over ∨”: (q ∨ r) ∧ p ≡ (q ∧ p) ∨ (r ∧ p)

Substitution

“Reflexivity of =”: x = x

⟨ Fact `10 - 3 = 7` ⟩

“Cancellation of +”: a + b = a + c ≡ b = c

Declaration: \_+\_ : ℤ → ℤ → ℤ

Declaration: \_·\_ : ℤ → ℤ → ℤ

Axiom (15.1) (15.1a) “Associativity of +”: (a + b) + c = a + (b + c)

Axiom (15.1) (15.1b) “Associativity of ·”: (a · b) · c = a · (b · c)

Axiom (15.2) (15.2a) “Symmetry of +”: a + b = b + a

Axiom (15.2) (15.2b) “Symmetry of ·”: a · b = b · a

Axiom (15.3) (15.3a) “Additive identity” “Identity of +”: 0 + a = a

Axiom (15.3) (15.3b) “Additive identity” “Identity of +”: a + 0 = a

Axiom (15.4) (15.4a) “Multiplicative identity” “Identity of ·”: 1 · a = a

Axiom (15.4) (15.4b) “Multiplicative identity” “Identity of ·”: a · 1 = a

Axiom (15.5) (15.5a) “Distributivity” “Distributivity of · over +”: a · (b + c) = a · b + a · c

Axiom (15.5) (15.5b) “Distributivity” “Distributivity of · over +”: (b + c) · a = b · a + c · a

Axiom (15.9) (15.9a) “Zero of ·”: a · 0 = 0

Axiom (15.9) (15.9b) “Zero of ·”: 0 · a = 0

Axiom “Definition of ≡”: (p ≡ q) = (p = q)

Axiom (3.1) “Associativity of ≡”: ((p ≡ q) ≡ r) ≡ (p ≡ (q ≡ r))

Axiom (3.2) “Symmetry of ≡”: p ≡ q ≡ q ≡ p

Axiom (3.3) “Identity of ≡”: true ≡ q ≡ q

Theorem (3.5) “Reflexivity of ≡”: p ≡ p

Axiom (3.8) “Definition of `false`”: false ≡ ¬ true

Axiom (3.9) “Commutativity of ¬ with ≡”: ¬(p ≡ q) ≡ ¬ p ≡ q